JILL A. HOLMAN

# GMM Estimation of a Money-in-the-Utility-Function Model: The Implications of Functional Forms

This paper studies consumer demand for real balances by allowing money to enter directly into an aggregate utility function as an asset that provides liquidity services. The essay extends existing literature by investigating a money-in-the-utility-function model under a variety of specifications of the representative agent's objective function. The empirical analysis employs the generalized-method-of-moments technique to estimate the coefficients of the Euler equations derived from the structural model. The parameter estimates are compared across specifications of the utility function and across data sets. The results provide some support for the view that the liquidity services provided by real balances contribute to utility.

THIS PAPER STUDIES CONSUMER DEMAND for real balances by allowing money to enter directly into an aggregate utility function as an asset that provides liquidity services.[1] The primary purpose of this work is to examine the benefits derived from money's services over time. This is accomplished by empirically investigating the Euler equations derived from a dynamic asset-pricing model in which a representative agent obtains utility from consumption and real balances.

Although many researchers have used a money-in-the-utility-function (MIUF) formulation for their empirical work on household demand for assets, the approach remains controversial.[2] One alternative is to have money enter an asset-pricing model via a cash-in-advance (CIA) constraint.[3] The CIA approach is quite popular in the literature, particularly in international finance models.[4] Feenstra (1986) demonstrates that, in many cases, the CIA formulation is theoretically equivalent to the MIUF ap-

The author thanks James Alm, Edward Balistreri, Charles de Bartolomé, JoAnne Feeney, Philip Graves, Robert McNown, Bárbara Robles, Martin Ross, Paul Wojick, and Jeffrey Zax for their comments on earlier drafts. Three anonymous referees provided suggestions that significantly improved the paper.

1. See McCallum and Goodfriend (1989) for a survey of theoretical work on the demand for real balances. See Goldfeld (1989) and Goldfeld and Sichel (1990) for an overview of empirical studies on money demand.

2. Chetty (1969), Feige and Pearce (1977), Calvo (1979), Ewis and Fisher (1984), Husted and Rush (1984), Poterba and Rotemberg (1987), and Koenig (1990) all employ the MIUF approach.

3. McCallum and Goodfriend (1989) develop a "shopping-time" model to motivate the demand for money. For another exposition of the shopping-time model and its connection to the log-log money demand function used frequently in empirical work, see Lucas (1994).

4. For example, see Lucas (1982) and all of its extensions in the literature.

JILL A. HOLMAN *is assistant professor of economics at the University of Georgia, Terry College of Business.*

*Journal of Money, Credit, and Banking,* Vol 30, No. 4 (November 1998)

proach.[5] Stockman (1989) points out that the CIA constraint captures only the transactions demand for money. The focus of the current paper is not to demonstrate that money mitigates transactions costs, but rather to explore the more general role of liquidity services in an agent's optimization problem. In addition to capturing transactions demand, placing real balances in the utility function allows for precautionary and store-of-value motives for holding money.

Other authors have examined asset-pricing models that include consumption and real money balances. Singleton (1985) investigates the relationships implied by the Lucas (1982) international finance model. Poterba and Rotemberg (1987) estimate the Euler equations of a model where several assets (including money) enter directly into the utility function. Finn, Hoffman, and Schlagenhauf (1990) empirically evaluate the asset-pricing relationships derived from alternative models to ascertain whether the inclusion of money helps to explain asset returns.

This paper provides two extensions of previous work. First, the Euler equations are estimated using the generalized-method-of-moments (GMM) technique under a variety of parameterizations of the utility function. Specifically, a Cobb-Douglas, a constant-elasticity-of-substitution (CES), and a nested-preference specification are employed. This approach permits a comparison of the results under alternative assumptions about the form of the utility function and extends the framework for testing MIUF formulations.

Second, the empirical work utilizes annual data from 1889 to 1991 on U.S. per capita money holdings, consumption, prices, and asset returns. Previous investigations in this area have been conducted with monthly or quarterly data spanning relatively few years. A data set that spans more than a century facilitates a comparison of previous findings to those that obtain over a longer time horizon. Also, a larger data set may improve the power of the econometric tests employed. Lengthy time series are only appropriate, however, if there are no relevant structural breaks in the data. The stability of the results for the full sample is examined by estimating the model with subsets of the complete time series. In addition, the robustness of the results is verified by using alternative measures of the return on money and the price level.

The estimates of the preference parameters are compared across the various forms of the utility function, across data sets, and to the estimates obtained in similar studies. Hansen's (1982) *J*-test of overidentifying restrictions is used to conduct a joint test of the characterization of the utility function and the validity of the instrument set used in the estimation. The *J*-test provides little guidance as to the appropriate specification of the utility function among all the alternatives estimated. Quasi-likelihood ratio tests are employed in an attempt to determine which characterization of the utility function best fits the data. The results lend some support to the notion that liquidity services significantly contribute to utility. Thus, this work complements that of Poterba and

5. LeRoy (1984) compares the MIUF formulation to alternative models and provides a defense for the MIUF approach as an heuristic device.

6. This model is similar to that in Poterba and Rotemberg (1987). The model differs from theirs in two respects. First, money is not assumed to be a "safe" asset in this model. Second, Poterba and Rotemberg use three assets (money (M1), short-term government debt, and real savings and time deposits) to capture liquidity's contribution to utility.

Rotemberg (1987) and provides some additional evidence in favor of the MIUF approach while employing a distinct estimation technique. The findings also indicate that more elaborate specifications of the utility function do not substantially improve the fit of the model. This result, however, may in part be attributed to the GMM estimator.

The organization of the paper is as follows. Section 1 develops the representative agent's utility-maximization problem and derives the Euler equations under alternative characterizations of the agent's utility function. Section 2 describes the data and the estimation procedure. The empirical results are presented in section 3. Brief conclusions are given in section 4.

## 1. THE MODEL

A representative consumer maximizes the expected discounted sum of utility over the infinite horizon by choosing consumption and real money balances subject to a sequence of budget constraints:

$$\text{Max } V_t = E_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} U\left(C_\tau, \frac{M_\tau}{P_\tau}\right)$$

$$\text{s.t.} \quad W_\tau = (1+r_{\tau-1}^W)W_{\tau-1} + (1+r_{\tau-1}^m)M_{\tau-1} - P_\tau C_\tau - M_\tau \tag{1}$$

where $E_t$ is the expectation operator conditional on the agent's information set at time $t$, $C_\tau$ is real consumption, $M_\tau/P_\tau$ is real money balances, $W_\tau$ denotes wealth remaining in $\tau - 1$ (after allocations to consumption expenditures and money holdings) that is invested in bonds, and $r^W$ and $r^m$ are the nominal one-period return on bonds and the nominal one-period return on real money holdings, respectively.[6] $\beta \in (0,1)$ is the discount factor, and $\beta = 1/(1+\rho)$ where $\rho$ is the rate of time preference. The representative agent has constant, additively time-separable preferences. It is assumed that these preferences are well behaved in that the instantaneous utility function $U(\cdot)$ is increasing at a decreasing rate in both arguments. The agent is assumed to have full current-period information.

Invoking the principle of optimality and the fundamental recursive relationship, the problem can be solved for any two periods $\tau = t$ and $\tau = t + 1$, and the solution will hold for all $t$ and $t + 1$:

$$V_t = Max_{C_t, M_t/P_t}\left[U\left(C_t, \frac{M_t}{P_t}\right)\right] + E_t\left[\beta U\left(C_{t+1}, \frac{M_{t+1}}{P_{t+1}}\right)\right]$$

$$\text{s.t.} \quad C_t = (1+r_{t-1}^W)\frac{W_{t-1}}{P_t} - \frac{W_t}{P_t} + (1+r_{t-1}^m)\frac{M_{t-1}}{P_t} - \frac{M_t}{P_t}$$

$$C_{t+1} = (1+r_t^W)\frac{W_t}{P_{t+1}} - \frac{W_{t+1}}{P_{t+1}} + (1+r_t^m)\frac{M_t}{P_{t+1}} - \frac{M_{t+1}}{P_{t+1}}. \tag{2}$$

Differentiating with respect to $C_t$ and $M_t/P_t$ and rearranging yields the following two Euler equations:

$$E_t\left[\beta\frac{\partial U_{t+1}/\partial C_{t+1}}{\partial U_t/\partial C_t}\frac{P_t}{P_{t+1}}(1+r_t^W)-1\right]=0, \tag{3}$$

$$E_t\left[\beta\frac{\partial U_{t+1}/\partial C_{t+1}}{\partial U_t/\left(\partial\dfrac{M_t}{P_t}\right)}\frac{P_t}{P_{t+1}}(r_t^W-r_t^m)-1\right]=0. \tag{4}$$

The Euler equation for consumption (3) states that, along an optimal path, the marginal cost of reducing consumption in $t$ by one unit is exactly equal to the expected, discounted, marginal benefit of investing the unit of consumption in bonds in $t$ and consuming the proceeds in $t + 1$. Similarly, equation (4) indicates that the marginal utility of a dollar held today must equal the discounted marginal value of next year's consumption of interest income were that dollar invested in a higher yielding bond.

It is assumed that the agent's utility function can take one of the following three functional forms:

Cobb-Douglas

$$U_t = C_t^{\alpha} m_t^{1-\alpha} \tag{5a}$$

CES

$$U_t=\left[\pi C_t^{\delta}+(1-\pi)m_t^{\delta}\right]^{\frac{1}{\delta}} \quad \text{for } \delta<1,\ \delta\neq 0 \tag{5b}$$

Nested CES

$$U_t=\frac{1}{\Psi}U(C_t,m_t)^{\Psi} \quad \text{for } \Psi<1,\ \Psi\neq 0 \tag{5c}$$

$$\text{or} \quad U_t=\ln U(C_t,m_t) \quad \text{for } \Psi=0$$

$$\text{where} \quad U(C_t,m_t)=\left[\omega C_t^{\gamma}+(1-\omega)m_t^{\gamma}\right]^{\frac{1}{\gamma}} \quad \text{for } \gamma<1,\ \gamma\neq 0$$

$$\text{or} \quad U(C_t,m_t)=C_t^{\omega}m_t^{1-\omega} \quad \text{for } \gamma=0$$

where $m_t = M_t/P_t$ for ease of exposition. These parameterizations were chosen because of their predominance in the asset-pricing literature. For example, Finn, Hoffman, and Schlagenhauf (1990) use a nested Cobb-Douglas in consumption and real balances, while Poterba and Rotemberg (1987) employ a nested-CES utility function in consumption and assets.

Utility function (5a) is Cobb-Douglas in consumption and real balances. This functional form assumes homogeneity, separability between its arguments and other sources of utility, imposes unitary elasticity of substitution between consumption and real balances, and displays constant relative risk aversion. Characterizations (5b) and (5c) have similar characteristics. The CES specification (5b) allows for estimation of the degree of intratemporal substitutability between consumption and real money balances, in contrast to the Cobb-Douglas specification. A nested CES, equation (5c), combines characterizations (5a) and (5b) and allows the coefficient of relative risk aversion and the intertemporal elasticity of substitution to be measured jointly.[7,8]

The Euler equations (3) and (4) under the alternative characterizations of the utility function become:

Cobb-Douglas

$$E_t\left[\beta\left(\frac{C_{t+1}}{C_t}\right)^{\alpha-1}\left(\frac{m_{t+1}}{m_t}\right)^{1-\alpha}\frac{P_t}{P_{t+1}}(1+r_t^W)-1\right]=0, \tag{3'a}$$

$$E_t\left[\beta\frac{\alpha}{1-\alpha}\left(\frac{m_{t+1}}{C_{t+1}}\right)^{1-\alpha}\left(\frac{m_t}{C_t}\right)^{\alpha}\frac{P_t}{P_{t+1}}(r_t^W-r_t^m)-1\right]=0, \tag{4'a}$$

CES

$$E_t\left[\beta\left(\frac{\pi C_{t+1}^{\delta}+(1-\pi)m_{t+1}^{\delta}}{\pi C_t^{\delta}+(1-\pi)m_t^{\delta}}\right)^{\frac{1-\delta}{\delta}}\left(\frac{C_{t+1}}{C_t}\right)^{\delta-1}\frac{P_t}{P_{t+1}}(1+r_t^W)-1\right]=0, \tag{3'b}$$

$$E_t\left[\beta\frac{\pi}{1-\pi}\left(\frac{\pi C_{t+1}^{\delta}+(1-\pi)m_{t+1}^{\delta}}{\pi C_t^{\delta}+(1-\pi)m_t^{\delta}}\right)^{\frac{1-\delta}{\delta}}\left(\frac{C_{t+1}}{m_t}\right)^{\delta-1}\frac{P_t}{P_{t+1}}(r_t^W-r_t^m)-1\right]=0, \tag{4'b}$$

Nested CES

$$E_t\left[\beta\left(\frac{\omega C_{t+1}^{\gamma}+(1-\omega)m_{t+1}^{\gamma}}{\omega C_t^{\gamma}+(1-\omega)m_t^{\gamma}}\right)^{\frac{\psi-\gamma}{\gamma}}\left(\frac{C_{t+1}}{C_t}\right)^{\gamma-1}\frac{P_t}{P_{t+1}}(1+r_t^W)-1\right]=0, \tag{3'c}$$

7. Under additively time-separable preferences, the coefficient of relative risk aversion is the inverse of the intertemporal elasticity of substitution. A nonexpected utility function would allow the two parameters to be estimated separately.

8. Note that the shares of consumption and money in expenditures for each utility function are constrained to be positive and to sum to unity.

$$E_t\left[\beta\frac{\omega}{1-\omega}\left(\frac{\omega C_{t+1}^{\gamma}+(1-\omega)m_{t+1}^{\gamma}}{\omega C_t^{\delta}+(1-\omega)m_t^{\gamma}}\right)^{\frac{\psi-\gamma}{\gamma}}\left(\frac{C_{t+1}}{m_t}\right)^{\gamma-1}\frac{P_t}{P_{t+1}}(r_t^W-r_t^m)-1\right]=0. \quad (4'c)$$

The parameters of the alternative utility functions are estimated by fitting the Euler equations to time series data.

## 2. ESTIMATION TECHNIQUE AND DATA

### *The GMM Estimation Technique*

The GMM technique, as described in Hansen (1982) and Hansen and Singleton (1982), is used to estimate each pair of Euler equations [(3′a)–(4′a) through (3′c)–(4′c)].[9] The two Euler equations for each parameterization of the utility function are estimated as a system. The intuition behind the GMM procedure is relatively simple. Dynamic optimization problems yield a set of stochastic Euler equations that must be satisfied in equilibrium. The Euler equations state that the representative agent's expectations are orthogonal to all of the variables in his/her information set at the time predictions are made.

The Euler equations imply population orthogonality conditions that depend on variables observed by the econometrician and on preference parameters. The GMM estimator is a nonlinear instrumental-variable estimator of the population parameters that makes the sample orthogonality conditions "close" to zero by minimizing a distance function. Hansen (1982) provides the conditions under which the GMM estimator is strongly consistent, asymptotically normal, and efficient in the class of all instrumental-variable estimators defined by orthogonality conditions.[10]

When there are more instruments than parameters, the system of Euler equations is overidentified. Hansen (1982) and Hansen and Singleton's (1982) *J*-test of overidentifying restrictions is used to conduct a joint test of the specification of the theoretical model and the validity of the instrument set. The *C*-test, as described in Eichenbaum, Hansen, and Singleton (1988), is employed to determine whether a unit value restriction on the share of expenditures devoted to consumption is appropriate.[11] In other words, *C*-tests are used to verify the validity of including liquidity services as mea-

9. For other applications of GMM in the asset-pricing context, see Eichenbaum, Hansen, and Singleton (1988), and Finn, Hoffman, and Schlagenhauf (1990). Davidson and MacKinnon (1993) and Gallant (1987) both provide quite readable theoretical expositions of GMM and the associated tests.

10. Stationarity of both the instrument set and the variables in the Euler equations is required for the GMM estimator to be strongly consistent, asymptotically normal, and efficient. Unit-root tests were conducted on the variables and the instruments used in the estimation. These results are available from the author by request.

11. *C*-tests are analogous to the quasi-likelihood ratio tests suggested by Gallant and Jorgenson (1979) for testing restrictions on parameters in the context of nonlinear instrumental-variable estimation techniques.

sured by real money balances directly in the utility function. Quasi-likelihood ratio tests are also utilized in an attempt to discern which of the functional forms is most appropriate for the data.

The residuals of the estimated Euler equations are examined for conditional heteroskedasticity and autocorrelation. The results are virtually unchanged when the standard errors are computed from a heteroskedastic consistent residual covariance matrix. Correcting for first- or second-order autocorrelation does not significantly change the results. This finding is robust to the choice of either the Parzen or the Bartlett spectral-density kernel. The reported results are those that obtain without any corrections.

*Caveats Associated with GMM*

A number of articles suggest that successful estimation with the GMM technique requires careful treatment of the instruments. Mao (1990) finds that the performance of the GMM estimator and the *J*-test are sensitive to the choice of the number of lags used in forming the vector of instruments. With fewer lagged instruments, the approximations of the objective function and the test statistic become more accurate. This result is consistent with Tauchen (1986), who urges that shorter lag lengths should be used to form the instrument vector.

Pagan (1998) reports that, in most studies, the *J*-test leads to a rejection of the theoretical model.[12] One possible reason for this, as noted by Mao (1990), is the poor sampling distribution of the GMM estimator. Pagan (1998) points out that an alternative explanation is misspecification of the objective function.[13] In contrast to other findings, Tauchen (1986) shows that the *J*-test performs well in moderately sized samples.

Fuhrer, Moore, and Schuh's (1995) Monte Carlo simulations reveal that GMM estimates are often biased, statistically insignificant, economically implausible (have the wrong signs), and dynamically unstable. As in Mao (1990), the authors suggest that the problems with the GMM estimator apparently hinge on the instruments used in the estimation technique. In particular, Fuhrer, Moore, and Schuh's (1995) report that the use of low-quality (weak, irrelevant) instruments can have deleterious effects on the GMM estimator. The authors state that a common-sense solution to the bias problem is to use lags of the variables that appear on the right-hand side of the regression as instruments. The lags are likely to be highly correlated with the right-hand side variables, and this choice of instruments should alleviate the problems associated with poor instrument relevance.

Steps were taken in this study to avoid the possible bias associated with the GMM estimator. Consistent with the suggestions of previous authors, the vector of instruments used to estimate each pair of Euler equations consists only of a constant and the variables entering into that set of equations. In addition, a single lag of instruments is used. The instrument set employed in each estimation is listed in the corresponding ta-

12. See Kocherlakota (1990c) also.
13. For more on this point, see Constantinides (1990).

bles. The instruments are similar to those of Singleton (1990) and Finn, Hoffman, and Schlagenhauf (1990). Nonetheless, the results indicate that the parameter estimates for the more complicated utility functions may be somewhat fragile.

*Data*

The alternative characterizations of equations (3) and (4) are estimated for the United States for the sample period 1889 to 1991. Real money balances are measured as annual M2 holdings deflated by the price level, where the price level is measured by both the consumer price index and the implicit GNP deflator.[14] All of these series were taken from Bordo and Jonung (1991).[15]

Consumption is measured as real (durable and nondurable) consumption expenditures in millions of 1972 dollars. The data from 1889 to 1896 are taken from Kendrick (1961), and for the period 1897 to 1928 are taken from DeLong and Summers (1986) as listed in Balke and Gordon (1986). The remainder of the consumption series is from the National Income and Product Accounts (NIPA). The nominal one-period return on bonds is proxied by the yield on corporate bonds. From 1889 to 1918, the data are from Macaulay (1938) as reported by Balke and Gordon (1986). For the rest of the sample, the measure is Moody's Baa rating.

Other empirical studies of this type have assumed that money is a "safe" asset; that is, money has zero risk, zero return, and is measured as M1. By contrast, this study employs M2, and thus it is necessary to include a measure of the "own" return on money holdings. In particular, it is necessary to obtain a measure of the spread between the return on bonds and the return on M2. Two proxies are used to capture the nominal return on real money balances. Klein's (1974, 1977) formula for the own-return on M2 is employed.[16,17] The short-term interest-rate series constructed by Bordo and Jonung (1991) is used as an alternative measure. The reason why the short-term interest-rate series may be considered an acceptable proxy is that the spread between the return on long-term corporate bonds and the short-term interest rate may be highly correlated with the return on bonds and the "true" return on M2.

## 3. EMPIRICAL RESULTS

*Cobb-Douglas Utility Function*

Table 1a presents the estimates of the parameters in equations (3′a) and (4′a) for the full sample.[18] The $J$-test of overidentifying restrictions easily indicates nonrejection

14. Poterba and Rotemberg (1987) employ quarterly data, whereas Finn, Hoffman, and Schlagenhauf (1990) and Singleton (1990) use monthly data.

15. See Bordo and Jonung (1991) for a complete description of the data.

16. Klein (1974) reports the return on M2 from 1880 to 1970. The data necessary to construct the 1971–1991 portion of the series were obtained from various issues of the *Federal Reserve Bulletin* and the *Statistics on Banking*.

17. See Belongia and Chalfant (1989) and Ladenson and Makinen (1992) for other examples of using Klein's (1974, 1977) formula.

18. The TSP (mainframe version 4.2b) algorithm for GMM was used to estimate the various characterizations of equations (3) and (4).

TABLE 1A

PARAMETER ESTIMATES OF EQUATIONS (3′A) AND (4′A)[a,b]
Cobb-Douglas Utility Function, 1894 to 1991

| Prices/Return | $\beta$ | SE($\beta$) | $\alpha$ | SE($\alpha$) | J(df) | C(df)[c] |
|---|---|---|---|---|---|---|
| DEFL/KLEIN | 0.9617** | 0.0058 | 0.9759** | 0.0011 | 0.4540 (3) | 38.91 (1) |
| DEFL/IS | 0.9623** | 0.0057 | 0.9854** | 0.0011 | 0.5145 (3) | 51.08 (1) |
| CPI/KLEIN | 0.9603** | 0.0051 | 0.9785** | 0.0010 | 0.4521 (3) | 36.91(1) |
| CPI/IS | 0.9596** | 0.0051 | 0.9827** | 0.0012 | 0.5921 (3) | 53.55 (1) |

**Significant at the 1 percent level.
*Significant at the 5 percent level.

[a]DEFL (CPI) denotes the implicit GNP deflator (consumer price index). KLEIN(IS) denotes Klein's (1974, 1977) formula [Bordo and Jonung's (1991) short-term interest rate] which is used to measure the return on M2 holdings. SE is the standard error of the corresponding parameter estimate. J(df) (C(df)) is the *J*-statistic (*C*-statistic) whose degrees of freedom are indicated in parentheses. Both the *J*-statistic and the *C*-statistic are distributed $\chi^2$.
[b]The instruments include a constant, $(C_{t-2}/C_{t-3})$, $(m_{t-2}/m_{t-3})$, $(r^W - r^m)_{t-2}$, and $(P_{t-3}/P_{t-2})$. Thus, five instruments are used to estimate two parameters, leaving three overidentifying restrictions to be tested. The $\chi^2$ statistic for three degrees of freedom at the 5 percent level is 7.81.
[c]The *C*-test is used to test the restriction that $\alpha = 1$. The $\chi^2$ statistic with one degree of freedom at the 5 percent level is 3.84.

of the MIUF model at the 5 percent significance level across all price and asset-return data. The estimated discount rate ($\beta$) is significantly greater than zero at the 1 percent level. The point estimate, however, is slightly smaller than that found in other studies using monthly or quarterly data.[19] The results provide some support for the view that real balances provide liquidity services that directly contribute to utility. The estimated share of expenditures devoted to consumption ($\alpha$) is significantly greater than zero at the 1 percent level and lies between 0.9759 and 0.9854. A *C*-test is used to determine whether $\alpha = 1$. If $\alpha$ were equal to one, liquidity services proxied by real money balances would not be a direct source of utility, and the data would be better explained by a simple barter model. As can be seen in Table 1a, the restriction $\alpha = 1$ is rejected by the data at all conventional significance levels.

As the time series employed in the estimation are quite lengthy, it is important to investigate the sensitivity of the results to potential breaks in the data. Tables 1b–1d display the parameter estimates when the start-date of the sample is changed to 1933, 1945, and 1955, respectively.[20] The findings are not dramatically different from those of the full sample. The role played by liquidity services in utility appears to be larger in the shorter samples. Moreover, the MIUF model cannot be rejected for any of the restricted samples.

To make the parameter estimates comparable to those in the standard empirical money-demand literature (for example, work on the log-log specification), "semi-interest elasticities" were computed following the procedure described in Poterba and Rotemberg (1987). Essentially, a semi-interest elasticity is the short-run response of

19. Hansen and Singleton (1983) estimate that $\beta$ lies between 0.995 and 1.096. Finn, Hoffman, and Schlagenhauf's (1990) and Singleton's (1990) empirical work indicate that the discount factor is close to or greater than one. Poterba and Rotemberg (1987) find that the discount rate is consistently larger than unity. See also Kocherlakota (1990a).

20. The sample was reduced to 1955–91 in order to compare the results to those found in Poterba and Rotemberg (1987) and Finn, Hoffman, and Schlagenhauf (1990). The sample was also reduced to 1920–1991, 1930–1991, 1940–1991, 1950–1991, 1889–1960, 1889–1950, 1889–1940, and 1889–1930. The results are very similar to those reported.

TABLE 1B

PARAMETER ESTIMATES OF EQUATIONS (3′A) AND (4′A)
Cobb-Douglas Utility Function, 1933 to 1991

| Prices/Return | β | SE(β) | α | SE(α) | J(df) |
|---|---|---|---|---|---|
| DEFL/KLEIN | 0.9695** | 0.0053 | 0.9707** | 0.0014 | 0.5383 (3) |
| DEFL/IS | 0.9713** | 0.0053 | 0.9778** | 0.0015 | 0.6154 (3) |
| CPI/KLEIN | 0.9677** | 0.0054 | 0.9734** | 0.0012 | 0.5072 (3) |
| CPI/IS | 0.9662** | 0.0054 | 0.9782** | 0.0015 | 0.6056 (3) |

Notes as for Table 1a.

TABLE 1C

PARAMETER ESTIMATES OF EQUATIONS (3′A) AND (4′A)
Cobb-Douglas Utility Function, 1945 to 1991

| Prices/Return | β | SE(β) | α | SE(α) | J(df) |
|---|---|---|---|---|---|
| DEFL/KLEIN | 0.9686** | 0.0059 | 0.9716** | 0.0016 | 0.6465 (3) |
| DEFL/IS | 0.9728** | 0.0059 | 0.9818** | 0.0018 | 0.5410 (3) |
| CPI/KLEIN | 0.9687** | 0.0058 | 0.9734** | 0.0017 | 0.6126 (3) |
| CPI/IS | 0.9680** | 0.0058 | 0.9798** | 0.0018 | 0.5953 (3) |

Notes as for Table 1a.

TABLE 1D

PARAMETER ESTIMATES OF EQUATIONS (3′A) AND (4′A)
Cobb-Douglas Utility Function, 1955 to 1991

| Prices/Return | β | SE(β) | α | SE(α) | J(df) |
|---|---|---|---|---|---|
| DEFL/KLEIN | 0.9565** | 0.0048 | 0.9698** | 0.0023 | 0.6909 (3) |
| DEFL/IS | 0.9620** | 0.0050 | 0.9816** | 0.0022 | 0.6345 (3) |
| CPI/KLEIN | 0.9584** | 0.0052 | 0.9719** | 0.0018 | 0.6782 (3) |
| CPI/IS | 0.9524** | 0.0052 | 0.9778** | 0.0023 | 0.7185 (3) |

Notes as for Table 1a.

money demand at time $t$ to a change in interest rates from $t$ to $t + 1$. Poterba and Rotemberg (1987) state that this is the closest analogue in this type of model to the interest-elasticity measures found in traditional money-demand studies. The demand for money [described by equation (4)] depends on the difference between the return on bonds and the return on money. Denote this spread as $u_m = (r_t^W - r_t^m)P_t/P_{t+1}$. The semi-interest elasticity of money demand is measured as the percentage change in money holdings in response to a percentage change in the spread. In other words, the semi-interest elasticity is calculated by differentiating equation (4) with respect to $u_m$.[21]

Table 2 reports the semi-interest elasticities for the Cobb-Douglas parameteriza-

21. During the calculations of the semi-interest elasticity, certainty equivalence is invoked. Consumption remains on its actual path (as described by the data), and the calculations use the value of $u_m$ that makes equation (4) hold with equality.

TABLE 2

SEMI-INTEREST ELASTICITIES OF MONEY DEMAND, COBB-DOUGLAS PARAMETERIZATION[a]

| Prices/Return | 1894–95[b] | 1950–51 | 1990–91 |
|---|---|---|---|
| DEFL/KLEIN | −0.200 | −0.681 | −0.372 |
| DEFL/IS | −0.341 | −0.397 | −0.638 |
| CPI/KLEIN | −0.158 | −0.414 | −0.295 |
| CPI/IS | −0.195 | −0.515 | −0.458 |

NOTES: [a]The calculations are based on the parameter estimates presented in Table 1a.
[b]The calculation of the interest elasticity requires data for periods $t$ and $t+1$. Here, the semi-interest elasticity is computed at three points in the sample: $t = 1894$ and $t+1 = 1895$; $t = 1950$ and $t+1 = 1951$; $t = 1990$ and $t+1 = 1991$.

tion using the parameter estimates for the full sample (Table 1a). The calculations were conducted at three points in the sample (1894–95, 1950–51, and 1990–91).[22] During the calculations, the spread was assumed to increase by one hundred basis points. As can be seen in Table 2, all the elasticities are of the correct sign; an increase in the spread decreases money holdings. All of the elasticities are of reasonable magnitude; however, some are smaller than those found by Poterba and Rotemberg (1987). Note that the semi-interest elasticities tend to be higher at later dates in the sample. This may be the result of the development of improved substitutes for money over time.

*CES Utility Function*

Table 3a shows the estimated parameters for equations (3′b) and (4′b) for the full sample. The *J*-test indicates that the MIUF model and the instruments employed in the estimation cannot be rejected at the 5 percent level for most of the data sets. When the GNP deflator and the short-term interest rate series are used, the theoretical specification of the model and the instrument set are jointly rejected. As can be seen in Tables 3b–3c, the failure of the MIUF model and the associated instruments with DEFL/IS is consistent across alternative sample sizes.

Under the CES parameterization, the estimated discount factor is significantly different from zero and is comparable to that obtained with the Cobb-Douglas parameterization. The share of expenditures on consumption ($\pi$) is slightly smaller than that indicated by the Cobb-Douglas estimates.[23] Thus, the CES specification permits a larger role for liquidity services as a direct source of utility over the full sample. The estimates of $\beta$ and $\pi$ for the periods 1933–1991 and 1945–1991 are shown in Tables 3b–3c and are similar to those found for the full sample.[24]

A priori, the substitution parameter ($\delta$) is expected to be less than unity and not equal to zero. If $\delta$ were exactly zero, the CES parameterization would degenerate to

22. Poterba and Rotemberg calculate the semi-interest elasticity for their parameter estimates using data from 1981:4 to 1982:1.

23. *C*-tests cannot be used to test the validity of the restriction $\pi = 1$. When this restriction is imposed, the marginal utility of consumption reduces to unity.

24. The findings for the period 1955–91 are not reported as the estimation procedure would not converge due to the small number of observations. The sample was also reduced to 1920–1991, 1930–1991, 1940–1991, 1950–1991, 1889–1960, 1889–1950, 1889–1940, and 1889–1930. The results are similar to those reported.

TABLE 3A

PARAMETER ESTIMATES OF EQUATIONS (3′B) AND (4′B)[a,b]
CES, 1894 to 1991

| Prices/Return | $\beta$ | SE($\beta$) | $\pi$ | SE($\pi$) | $\delta$ | SE($\delta$) | J(df) |
|---|---|---|---|---|---|---|---|
| DEFL/KLEIN | 0.9597** | 0.0058 | 0.9693** | 0.0020 | 0.8959** | 0.1776 | 0.67 (3) |
| DEFL/IS | 0.9609** | 0.0058 | 0.9761** | 0.0027 | 0.8234* | 0.3300 | 64.7 (3) |
| CPI/KLEIN | 0.9569** | 0.0051 | 0.9684** | 0.0023 | 0.9726** | 0.1337 | 0.75 (3) |
| CPI/IS | 0.9574** | 0.0051 | 0.9746** | 0.0032 | 0.9879** | 0.2576 | 0.76 (3) |

[a]Notes as for Table 1a.
[b]The instruments include a constant, $(C_{t-2}/C_{t-3})$, $(C_{t-2}/m_{t-2})$, $((0.95C_{t-2} + 0.05m_{t-2})/(0.95C_{t-3} + 0.05\ m_{t-3}))$, $(r^W - r^m)_{t-2}$, and $(P_{t-3}/P_{t-2})$. I am grateful to an anonymous referee for help in selecting these instruments. Six instruments are used to estimate three parameters, leaving three overidentifying restrictions to be tested. The $\chi^2$ statistic for three degrees of freedom at the 5 percent level is 7.81.

TABLE 3B

PARAMETER ESTIMATES OF EQUATIONS (3′B) AND (4′B)
CES, 1933 to 1991

| Prices/Return | $\beta$ | SE($\beta$) | $\pi$ | SE($\pi$) | $\delta$ | SE($\delta$) | J(df) |
|---|---|---|---|---|---|---|---|
| DEFL/KLEIN | 0.9698** | 0.0054 | 0.9718** | 0.0015 | −0.3559 | 0.2753 | 0.51 (3) |
| DEFL/IS | 0.9700** | 0.0054 | 0.9785** | 0.0018 | −0.7117 | 0.4212 | 29.7 (3) |
| CPI/KLEIN | 0.9673** | 0.0054 | 0.9735** | 0.0015 | −0.0545 | 0.1923 | 0.56 (3) |
| CPI/IS | 0.9673** | 0.0054 | 0.9806** | 0.0018 | −0.4111 | 0.3185 | 0.56 (3) |

Notes as for Table 2a.

TABLE 3C

PARAMETER ESTIMATES OF EQUATIONS (3′B) AND (4′B)
CES, 1945 to 1991

| Prices/Return | $\beta$ | SE($\beta$) | $\pi$ | SE($\pi$) | $\delta$ | SE($\delta$) | J(df) |
|---|---|---|---|---|---|---|---|
| DEFL/KLEIN | 0.9681** | 0.0058 | 0.9713** | 0.0018 | 0.5270 | 1.2331 | 0.85 (3) |
| DEFL/IS | 0.9670** | 0.0057 | 0.9774** | 0.0023 | 2.0199 | 1.8734 | 39.1(3) |
| CPI/KLEIN | 0.9703** | 0.0059 | 0.9758** | 0.0020 | −1.1047 | 0.8817 | 0.68 (3) |
| CPI/IS | 0.9694** | 0.0059 | 0.9820** | 0.0027 | −1.2101 | 1.5782 | 0.72 (3) |

Notes as for Table 2a.

the Cobb-Douglas specification of the utility function. Table 3a reveals that for the full sample the estimated coefficient is significantly different from zero at the 5 percent level in all of the cases examined. By contrast, Tables 3b–3c show that $\delta$ is insignificant in the shorter samples.[25]

In an attempt to shed light on the insignificant estimates of the degree of substitution between consumption and money holdings, quasi-likelihood tests are used to investigate whether the Cobb-Douglas ($\delta = 0$) restriction more accurately reflects the

25. Semi-interest elasticities could not be calculated for the CES specification. For the full sample, the interest elasticities explode because $\delta$ is very close to the edge of concave parameter space. For the shorter samples, the interest elasticities are nonsensical as $\delta$ is statistically equal to zero.

TABLE 4

Quasi-Likelihood Ratio Tests,[a,b] CES Parameterization
$H_0: \delta = 0$ (Cobb-Douglas)
$H_1: \delta < 1, \delta \neq 0$ (CES)

| Prices | Return | Test Statistic[c] | Conclusion[d] |
|---|---|---|---|
| DEFL | KLEIN | 0.09755 | cannot reject $H_0$ |
| DEFL | IS | −0.00436 | N/A |
| CPI | KLEIN | 0.05987 | cannot reject $H_0$ |
| CPI | IS | 0.02165 | cannot reject $H_0$ |

[a]Notes as for Table 1a.
[b]The instruments include a constant, $(C_{t-2}/C_{t-3})$, $(C_{t-2}/m_{t-2})$, $((0.95C_{t-2} + 0.05m_{t-2})/(0.95C_{t-3} + 0.05m_{t-3}))$, $(r^W - r^m)_{t-2}$, and $(P_{t-3}/P_{t-2})$. The instrument set and the estimated covariance of the disturbances are held constant across the restricted and unrestricted models.
[c]The test statistic is $T = n(Q_0 - Q_1)$ where $n$ is the number of observations, $Q_0$ denotes the minimum distance criterion under $H_0$, and $Q_1$ is the minimum distance criterion under $H_1$. The $T$-statistic is distributed $\chi^2$ with degrees of freedom equal to the difference in the number of parameters in the two models.
[d]The $\chi^2$ statistic with one degree of freedom at the 5 percent significance level is 3.84.

data than the CES characterization ($\delta \neq 0$).[26] During the testing procedure, the CES instrument set and the estimated covariance matrix are held constant across the two characterizations of the utility function. The results of the tests are displayed in Table 4. The Cobb-Douglas restriction ($\delta = 0$) cannot be rejected in most of the cases examined.[27]

*Nested Utility Function*

The intertemporal elasticity of substitution ($\psi$) is expected to be less than one and not equal to zero. If $\psi = 0$, then the nested function degenerates to a logarithmic function of consumption and real balances. The intratemporal elasticity of substitution between money and real balances is captured by $1/(1 - \gamma)$. A priori, $\gamma$ should be less than unity. If $\gamma$ were equal to zero, the form of the utility function reduces to the nested-Cobb-Douglas specification. Thus, four parameterizations of equations (3′c) and (4′c) are estimated: a nested CES, a nested Cobb-Douglas, a logarithmic CES, and a logarithmic Cobb-Douglas.

The estimated parameters for the full sample (1894–1991) are shown in Table 5.[28] The $J$-test easily indicates nonrejection of the money-in-the-utility-function model and the instrument sets employed in the GMM procedure at the 5 percent significance

26. The testing procedure is as follows. First, the CES parameterization is estimated using 2SLS to obtain a consistent estimate ($S$) of the covariance matrix. Second, $S$ and the CES instrument set are used to estimate the Cobb-Douglas parameterization with GMM. Third, $S$ and the CES instrument matrix are used to estimate the CES parameterization with GMM. Finally, the minimum-distance function from step 3 is subtracted from that of step 2. The test statistic is distributed asymptotically $\chi^2$ with degrees of freedom equal to the differences in the number of parameters in the restricted and unrestricted models.

27. The test statistic is ill-behaved under the DEFL/IS data set, which is consistent with the failure of the MIUF model when this data set is employed.

28. The sample was also reduced to 1920–1991, 1930–1991, 1933–1991, 1940–1991, 1945–1991, 1950–1991, 1955–1991, 1889–1960, 1889–1950, 1889–1945, 1889–1940, 1889–1930, and 1889–1925. The results for the restricted samples are not included either because of their similarity to the results reported or because the reduction in the number of observations resulted in a proliferation of arithmetic errors and caused the GMM procedure to fail to converge.

TABLE 5

PARAMETER ESTIMATES OF EQUATIONS (3′C) AND (4′C)[a]
Nested Characterization of the Utility Function, 1894 to 1991

| | Nested CES[b] | | | |
|---|---|---|---|---|
| | DEFL. KLEIN | DEFL. IS | CPI, KLEIN | CPI, IS |
| β | 0.9477** | 0.9474** | 0.9394** | 0.9383** |
| SE(β) | 0.0123 | 0.0138 | 0.0117 | 0.0134 |
| ψ | 1.6013** | 1.6609* | 1.8615** | 1.9147** |
| SE(ψ) | 0.5816 | 0.6714 | 0.5484 | 0.6447 |
| ω | 0.9691** | 0.9758** | 0.9681** | 0.9744** |
| SE(ω) | 0.0020 | 0.0026 | 0.0022 | 0.0032 |
| γ | 0.9308** | 0.8509* | 0.9616** | 0.9893** |
| SE(γ) | 0.1746 | 0.3246 | 0.1306 | 0.2514 |
| J(df) | 0.69(3) | 0.69(3) | 0.73(3) | 0.74(3) |
| C(df) | 55.76(1) | 56.47(1) | 58.56(1) | 58.56(1) |
| | **Nested Cobb-Douglas[c]** | | | |
| | DEFL. KLEIN | DEFL. IS | CPI, KLEIN | CPI, IS |
| β | 0.9423** | 0.9326** | 0.9361** | 0.9239** |
| SE(β) | 0.0144 | 0.0196 | 0.0142 | 0.0204 |
| ψ | 1.8824** | 2.3478* | 2.0551** | 2.5872** |
| SE(ψ) | 0.6935 | 0.9608 | 0.6818 | 0.9966 |
| ω | 0.9746** | 0.9799** | 0.9768** | 0.9817** |
| SE(ω) | 0.0012 | 0.0014 | 0.0011 | 0.0013 |
| J(df) | 0.71(3) | 0.65(3) | 0.79(3) | 0.69(3) |
| C(df) | 61.04(1) | 58.02(1) | 68.73(1) | 61.55(1) |
| | **Logarithmic CES[d]** | | | |
| | DEFL. KLEIN | DEFL. IS | CPI, KLEIN | CPI, IS |
| β | 0.9808** | 0.9815** | 0.9788** | 0.9796** |
| SE(β) | 0.0072 | 0.0072 | 0.0065 | 0.0065 |
| ω | 0.9793** | 0.9843** | 0.9790** | 0.9854** |
| SE(ω) | 0.0023 | 0.0021 | 0.0025 | 0.0024 |
| γ | −0.7019 | −0.9809 | −0.0582 | −0.5807 |
| SE(γ) | 0.5446 | 0.6881 | 0.3487 | 0.5257 |
| J(df) | 0.34(3) | 0.44(3) | 0.47(3) | 0.52(3) |
| C(df) | 24.30(1) | 33.15(1) | 36.06(1) | 40.16(1) |
| | **Logarithmic Cobb-Douglas[e]** | | | |
| | DEFL. KLEIN | DEFL. IS | CPI, KLEIN | CPI, IS |
| β | 0.9790** | 0.9802** | 0.9779** | 0.9782** |
| SE(β) | 0.0072 | 0.0072 | 0.0065 | 0.0065 |
| ω | 0.9751** | 0.9804** | 0.9774** | 0.9824** |
| SE(ω) | 0.0012 | 0.0013 | 0.0013 | 0.0012 |
| J(df) | 0.66(3) | 0.63(3) | 0.74(3) | 0.69(3) |
| C(df) | 56.09(1) | 52.22(1) | 63.64(1) | 58.39(1) |

[a]Notes as for Table 1a except the C-test is now used to test the restriction $\omega = 1$.

[b]The instruments include a constant, $(C_{t-2}/C_{t-3})$, $(m_{t-2}/m_{t-3})$, $(C_{t-2}/m_{t-2})$, $((0.95C_{t-2} + 0.05m_{t-2})/(0.95C_{t-3} + 0.05m_{t-3})$, $(r^W - r^m)_{t-2}$, and $(P_{t-3}/P_{t-2})$. I am grateful to an anonymous referee for help in selecting these instruments.

[c]The instruments include a constant, $(C_{t-2}/C_{t-3})$, $(m_{t-2}/m_{t-3})$, $(C_{t-2}/m_{t-2})$, $(r^W - r^m)_{t-2}$, and $(P_{t-3}/P_{t-2})$.

[d]The instruments include a constant, $(C_{t-2}/C_{t-3})$, $(m_{t-2}/m_{t-3})$, $((0.95C_{t-2} + 0.05m_{t-2})/(0.95C_{t-3} + 0.05m_{t-3})$, $(r^W - r^m)_{t-2}$, and $(P_{t-3}/P_{t-2})$.

[e]The instruments include a constant, $(C_{t-2}/C_{t-3})$, $(C_{t-2}/m_{t-2})$, $(r^W - r^m)_{t-2}$, and $(P_{t-3}/P_{t-2})$.

level for each of the four parametric representations of the utility function and across each of the four data sets.

The results suggest that liquidity services, as proxied by real money balances, play a role in utility. The share of expenditures devoted to consumption ($\omega$) is significantly different from zero at the 1 percent significance level in all cases. The share of expenditures on real balances ($1 - \omega$) is most accurately measured in the Cobb-Douglas characterizations. All estimates, however, have small standard errors. Liquidity services have the largest role in the nested-CES case [($1 - \omega$) ranges from 0.0242 to 0.0319] and the smallest role in the logarithmic-CES case (0.0146 to 0.0210). The estimates here indicate a comparable role for money balances relative to previous studies. Finn, Hoffman, and Schlagenhauf (1990) also find that real balances comprise less than 10 percent of total expenditures. Poterba and Rotemberg (1987) estimate that the share of expenditures on consumption is between 0.961 and 0.969. Thus, their estimates reveal a slightly larger role for liquidity services.

To buttress the finding that liquidity services contribute to utility, *C*-tests are conducted to determine whether the data are more accurately reflected by a barter model than a MIUF model. In other words, the appropriateness of the restriction $\omega = 1$ is tested. The results for the full sample are presented in Table 5. The restriction (liquidity services are not a source of utility) is rejected by the data across all characterizations of the utility function and with all of the alternative price and interest rate measures.

Some of the estimated parameters for the nested characterization are outside the concave region or are insignificantly different from zero.[29] Table 5 reveals that the intertemporal elasticity of substitution ($\psi$) is outside concave parameter space in all cases. Finn, Hoffman, and Schlagenhauf (1990) find $\psi$ to be insignificant in some specifications of the MIUF model and positive (less than one) in others. By contrast, Poterba and Rotemberg (1987) note that early estimates of $\psi$ range between $-6.0$ and $-0.8$, and their estimates lie between $-6.20$ and $-5.60$. Table 5 also indicates the estimated exponent of the CES characterizations ($\gamma$) is statistically different from zero in the nested-CES case but is insignificant in the logarithmic-CES case.

Testing the restrictions implied by the various parameterizations embedded in equations (3′c) and (4′c) may shed light on the estimates found in Table 5. Quasi-likelihood ratio tests are used to determine which of the restrictions are appropriate for the data for the full sample. Throughout the testing procedure, the nested-CES parameterization is the maintained hypothesis. Hence, the nested-CES instrument set and estimated covariance of disturbances are held constant across all of the tests. The results are shown in Table 6.

Three separate null hypotheses are entertained. First, the logarithmic CES is tested against the maintained hypothesis. The logarithmic restriction ($\psi = 0$) is supported in all of the cases examined. Second, the nested Cobb-Douglas is compared to the nested CES. The null hypothesis cannot be rejected for any of the data sets. Finally, the logarithmic Cobb-Douglas is tested against the nested CES. The restrictions ($\psi = 0$

29. Because $\psi$ is outside of concave parameter space, semi-interest elasticities cannot be calculated for this parameterization of the utility function.

TABLE 6

QUASI-LIKELIHOOD RATIO TESTS, NESTED CHARACTERIZATION OF THE UTILITY FUNCTION[a,b]

$H_0$: $\psi = 0$ and $\gamma < 1, \gamma \neq 0$ (Logarithmic CES)
$H_1$: $\psi < 1, \psi \neq 0$ and $\gamma < 1, \gamma \neq 0$ (Nested CES)

| Prices | Return | Test Statistic[c] | Conclusion[d] |
|---|---|---|---|
| DEFL | KLEIN | 0.04130 | cannot reject $H_0$ |
| DEFL | IS | 0.04427 | cannot reject $H_0$ |
| CPI | KLEIN | 0.07938 | cannot reject $H_0$ |
| CPI | IS | 0.07594 | cannot reject $H_0$ |

$H_0$: $\psi < 1$, $\psi \neq 0$ and $\gamma = 0$ (Nested Cobb-Douglas)
$H_1$: $\psi < 1$, $\psi \neq 0$ and $\gamma < 1, \gamma \neq 0$ (Nested CES)

| Prices | Return | Test Statistic[c] | Conclusion[d] |
|---|---|---|---|
| DEFL | KLEIN | 0.38737 | cannot reject $H_0$ |
| DEFL | IS | 0.38155 | cannot reject $H_0$ |
| CPI | KLEIN | 0.35617 | cannot reject $H_0$ |
| CPI | IS | 0.34316 | cannot reject $H_0$ |

$H_0$: $\psi = 0$ and $\gamma = 0$ (Logarithmic Cobb-Douglas)
$H_1$: $\psi < 1, \psi \neq 0$ and $\gamma < 1, \gamma \neq 0$ (Nested CES)

| Prices | Return | Test Statistic[c] | Conclusion[d] |
|---|---|---|---|
| DEFL | KLEIN | 0.10462 | cannot reject $H_0$ |
| DEFL | IS | 0.03164 | cannot reject $H_0$ |
| CPI | KLEIN | 0.11012 | cannot reject $H_0$ |
| CPI | IS | 0.05629 | cannot reject $H_0$ |

[a]Notes as for Table 1a.
[b]The instruments are those used to estimate the nested-CES utility function. See Table 5 note b.
[c]The test statistic is $T = n(Q_0 - Q_1)$ where $n$ is the number of observations, $Q_0$ denotes the minimum distance criterion under $H_0$, and $Q_1$ is the minimum distance criterion under $H_1$.
[d]The $\chi^2$ statistic at the 5 percent significance level for one degree of freedom is 3.84 and for two degrees of freedom is 5.99.

and $\gamma = 0$) cannot be rejected at the 5 percent level, which indicates that the logarithmic Cobb-Douglas reflects the data more accurately than does the nested CES. Moreover, the validity of the restrictions implied by the three null hypotheses supports the insignificance of the parameter estimates in Tables 3 and 5. Surprisingly, it appears that the more restrictive functional forms may be appropriate for asset-pricing models of this type.

Several other authors also report that the estimate of the parameter governing the intertemporal elasticity of substitution is difficult to pinpoint in empirical studies.[30] One reason may be the restriction that preferences are additively time separable.[31] That is, these results may indicate that a nonexpected utility function is appropriate. Mao (1990) suggests that in small sample sizes, and when the true value of the intertemporal elasticity of substitution is small, the GMM procedure tends to underestimate the intertemporal elasticity. Tauchen (1986) also finds that the GMM estimate of the intertemporal elasticity of substitution can be biased. Mao's (1990) results indi-

30. See Eichenbaum, Hansen, and Singleton (1988), Singleton (1985), and Hansen and Singleton (1982, 1983, and 1996).

31. For a discussion of the use of nonexpected utility functions in studies of intertemporal consumption decisions, see Epstein (1988) and Obstfeld (1990). See also Kocherlakota (1990b).

cate that the bias is reduced as the sample size increases, and more importantly, the magnitude of the bias might not be quantitatively important even in small samples. Mao concludes that the intertemporal elasticity of substitution can be adequately estimated using the GMM procedure, and failure to obtain an accurate measure of this parameter may result from measurement errors that are often present in consumption data.

Steps were taken to avoid the pitfalls of using the GMM technique. It does not appear that this study is plagued by the over-rejection of the theoretical model as found by Mao. For the majority of the parameters examined, the estimates accord well with the underlying theory. In some cases, however, the estimates obtained with the GMM estimator are theoretically unpalatable and/or insignificantly different from zero.

## 4. CONCLUSIONS

This paper presents a dynamic model of aggregate demand for real balances and consumption where money holdings enter directly into the representative agent's utility function. The model implies a set of Euler equations that govern the agent's choices. These Euler equations are empirically investigated using the GMM technique, and the results are verified with a variety of testing procedures. The primary contribution of the paper is to expand the framework for investigating the MIUF model.

This study extends previous work in several ways. First, the Euler equations are estimated under alternative parametric representations of the model. Second, annual data spanning a century are utilized. By contrast, most asset-pricing studies of this type employ monthly or quarterly data covering a much shorter time interval. Additionally, different measures of the price level and proxies for the return on real balances are used. Therefore, in addition to comparing the results to those already in the literature, the findings in this paper can be compared across the different specifications of the utility function and across data sets.

The parameter estimates are relatively robust to the choice of data and to the sample size, although in some cases they differ from those already in the literature. The intertemporal elasticity of substitution and the CES substitution parameter are often insignificant or outside the concave region of parameter space. Quasi-likelihood ratio tests reveal that the data support constraining the intertemporal elasticity to be equal to zero and the intratemporal elasticity of substitution to unity. Thus, it appears that the more restrictive functional forms may accurately reflect the data over this time span. However, this finding could be associated with the use of additively time-separable preferences or with the use of the GMM estimation technique.

The results lend some support to the view that real balances provide valued services that significantly contribute to the agent's utility flow. This result obtains across most of the parameterizations of the model and across most data sets. Additionally, the MIUF (and the validity of the instrument set) cannot be rejected in the majority of the cases examined.

## LITERATURE CITED


Balke, Nathan S., and Robert J. Gordon. "Historical Data." In *The American Business Cycle: Continuity and Change,* edited by R. J. Gordon. Chicago: University of Chicago Press, 1986.

Belongia, Michael T., and James A. Chalfant. "The Changing Empirical Definition of Money: Some Estimates from a Model of the Demand for Money Substitutes." *Journal of Political Economy* 97 (April 1989), 386–97.

Board of Governors of the Federal Reserve System. *The Federal Reserve Bulletin.* Washington, D.C., various issues, 1971–91.

Bordo, Michael D., and Lars Jonung. "The Bretton Woods International Monetary System: An Historical Overview." Paper presented at the University of Colorado at Boulder, 1991.

Calvo, Guillermo A. "On Models of Money and Perfect Foresight." *International Economic Review* 20 (February 1979), 83–103.

Chetty, V. Karuppan. "On Measuring the Nearness of Near-Moneys." *American Economic Review* 59 (June 1969), 270–81.

Constantinides, George M. "Habit Formation: A Resolution of the Equity Premium Puzzle." *Journal of Political Economy* 98 (June 1990), 519–43.

Davidson, Russell, and James G. MacKinnon. *Estimation and Inference in Econometrics.* New York: Oxford University Press, 1993.

DeLong, J. Bradley, and Lawrence H. Summers. "The Changing Cyclical Variability of Economic Activity in the United States." In *The American Business Cycle: Continuity and Change,* edited by R. J. Gordon. Chicago: University of Chicago Press, 1986.

Eichenbaum, Martin S., Lars P. Hansen, and Kenneth J. Singleton. "A Time Series Analysis of Representative Agent Models of Consumption and Leisure Choice under Uncertainty." *The Quarterly Journal of Economics* 103 (February 1988), 51–78.

Epstein, Larry G. "Risk Aversion and Asset Prices." *Journal of Monetary Economics* 22 (September 1988), 179–92.

Ewis, Nabil A., and Douglas Fisher. "The Translog Utility Function and the Demand for Money in the United States." *Journal of Money, Credit, and Banking* 16 (February 1984), 34–52.

Federal Deposit Insurance Corporation. *Statistics on Banking.* Washington, D.C., various issues, 1971–1991.

Feenstra, Robert C. "Functional Equivalence between Liquidity Costs and the Utility of Money." *Journal of Monetary Economics* 17 (March 1986), 271–91.

Feige, Edgar L., and Douglas K. Pearce. "The Substitutability of Money and Near-Moneys: A Survey of the Time Series Evidence." *Journal of Economic Literature* 15 (June 1977), 439–69.

Finn, Mary G., Dennis L. Hoffman, and Don E. Schlagenhauf. "Intertemporal Asset-Pricing Relationships in Barter and Monetary Economies." *Journal of Monetary Economics* 25 (June 1990), 431–51.

Fuhrer, Jeffrey C., George R. Moore, and Scott D. Schuh. "Estimating the Linear-Quadratic Inventory Model: Maximum Likelihood versus Generalized Method of Moments." *Journal of Monetary Economics* 35 (February 1995), 115–57.

Gallant, Ronald A. *Nonlinear Statistical Methods.* New York: John Wiley and Sons, 1987.

Gallant, Ronald A., and Dale W. Jorgenson. "Statistical Inference for a System of Simultaneous, Nonlinear, Implicit Equations in the Context of Instrumental Variable Estimation." *Journal of Econometrics* 11 (October/December 1979), 275–302.

Goldfeld, Stephen M. "Demand for Money: Empirical Studies." In *The New Palgrave: Money,* edited by J. Eatwell, M. Milgate, and P. Newman. New York: W. W. Norton, 1989.

Goldfeld, Stephen M., and Daniel E. Sichel. "The Demand for Money." In *Handbook of Mone-*

*tary Economics*, vol. I, edited by B. M. Friedman and F. H. Hahn. New York: Elsevier Science Publishers, 1990.

Hansen, Lars P. "Large Sample Properties of Generalized Methods of Moments Estimators." *Econometrica* 40 (July 1982), 1029–54.

Hansen, Lars P., and Kenneth J. Singleton. "Generalized Instrumental Variables Estimation of Nonlinear Rational Expectations Models." *Econometrica* 50 (September 1982), 1269–86.

______. "Stochastic Consumption Risk and the Temporal Behavior of Asset Returns." *Journal of Political Economy* 91 (April 1983), 249–65.

______. "Efficient Estimation of Linear Asset Pricing Models with Moving Average Errors." *Journal of Business and Economic Statistics* 14 (January 1996), 53–68.

Husted, Steven L., and Mark Rush. "On Measuring the Nearness of Near Monies: Revisited." *Journal of Monetary Economics* 14 (September 1984), 171–81.

Kendrick, John W. *Productivity Trends in the United States.* New York: National Bureau of Economic Research, 1961.

Klein, Benjamin J. "Competitive Interest Payments on Bank Deposits and the Long-Run Demand for Money." *American Economic Review* 64 (December 1974), 931–49.

______. "The Demand for Quality-Adjusted Cash Balances: Price Uncertainty in the U.S. Demand for Money Function." *Journal of Political Economy* 85 (August 1977), 691–715.

Kocherlakota, Narayana R. "On the 'Discount' Factor in Growth Economies." *Journal of Monetary Economics* 25 (January 1990), 43–47 (a).

______. "Disentangling the Coefficient of Relative Risk Aversion from the Elasticity of Intertemporal Substitution: An Irrelevance Result." *Journal of Finance* 45 (March 1990), 175–90 (b).

______. "On Tests of Representative Consumer Asset Pricing Models." *Journal of Monetary Economics* 26 (October 1990), 285–304 (c).

Koenig, Evan F. "Real Money Balances and the Timing of Consumption." *The Quarterly Journal of Economics* 105 (May 1990), 399–425.

Ladenson, Mark L., and Gail E. Makinen. "The Currency Ratios 1920–80: A Re-examination." *Atlantic Economic Journal* 20 (December 1992), 1–9.

LeRoy, Stephen F. "Nominal Prices and Interest Rates in General Equilibrium: Money Shocks." *Journal of Business* 57 (April 1984), 177–95.

Lucas, Robert E., Jr. "Interest Rate and Currency Prices in a Two-Country World." *Journal of Monetary Economics* 10 (November 1982), 335–60.

______. "On the Welfare Cost of Inflation." Working paper #94-07 at the Federal Reserve Bank of San Francisco, 1994.

Macaulay, Fredrick R. "The Movements of Interest Rates, Bonds, Yields, and Stock Prices in the U.S. since 1856." New York: National Bureau of Economic Research, 1938.

Mao, Ching-Sheng. "Hypothesis Testing and Finite Sample Properties of Generalized Method of Moments Estimators: A Monte Carlo Study." Working paper 90-12 at the Federal Reserve Bank of Richmond, 1990.

McCallum, Bennett T., and Marvin S. Goodfriend. "Demand for Money: Theoretical Studies." In *The New Palgrave: Money*, edited by J. Eatwell, M. Milgate, and P. Newman. New York: W. W. Norton, 1989.

Obstfeld, Maurice. "Intertemporal Dependence, Impatience, and Dynamics." *Journal of Monetary Economics* 26 (August 1990), 45–75.

Pagan, Adrian. "The Econometrics of Financial Markets." *Journal of Empirical Finance*, forthcoming.

Poterba, James M., and Julio J. Rotemberg. "Money in the Utility Function: An Empirical Implementation." In *New Approaches to Monetary Economics: Proceedings of the Second International Symposium in Economic Theory and Econometrics,* edited by W. Barnett and K. Singleton. Cambridge: Cambridge University Press, 1987.

Singleton, Kenneth J. "Testing Specifications of Economic Agent's Intertemporal Optimum Problems in the Presence of Alternative Models." *Journal of Econometrics* 30 (October/November 1985), 391–413.

______. "Specification and Estimation of Intertemporal Asset Pricing Models." In *Handbook of Monetary Economics, vol. I,* edited by B. M. Friedman and F. H. Hahn. New York: Elsevier Publishing, 1990.

Stockman, Alan C. "The Cash-in-Advance Constraint in International Economics." In *Finance Constraints and the Theory of Money,* edited by Meir Kohn. San Diego; London; Sydney; and Toronto: Harcourt, Brace, and Jovanovich Academic Press, 1989.

Tauchen, George. "Statistical Properties of Generalized Method-of-Moments Estimators of Structural Parameters Obtained from Financial Market Data." *Journal of Business and Economic Statistics* 4 (October 1986), 397–416.